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Machine Learning-aided Optimal Control of a Qubit under non-Markovian dynamics

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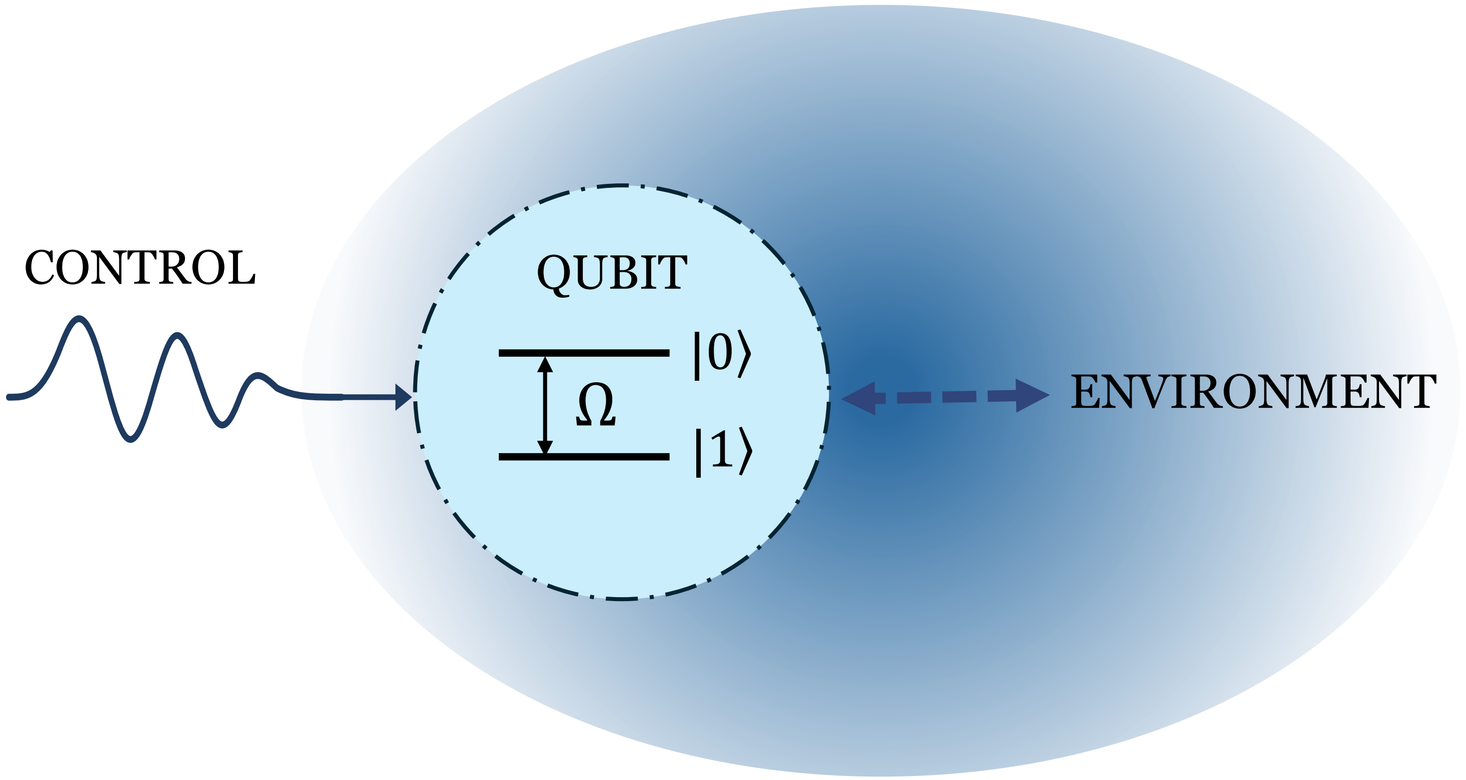
Control of open quantum systems

The challenge

- Qubits interact with control and environment.
- Non-Markovian dynamics: memory effects render standard approaches insufficient.
- Environmental noise causes decoherence.
- Quantum gates require precise control pulses.



Control of open quantum systems

Whitebox

Pros:

- Rigorous.
- Interpretable.

Cons:

- For open systems, it often requires strong assumptions and/or approximations.
- Hard to model non-Markovian dynamics.

E.g.

$$\dot{\rho} = -i[H(t), \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right)$$

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Blackbox

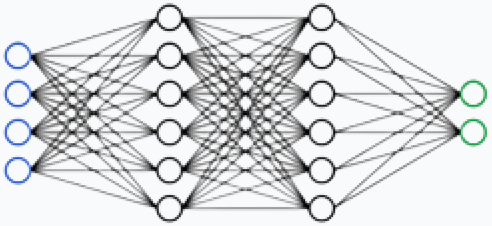
Pros:

- Doesn't require any a priori hypothesis.
- Highly flexible.

Cons:

- Lacks interpretability.
- Often requires large training datasets.

E.g.



Control of open quantum systems

Whitebox

Blackbox

Graybox

- Rigor and interpretability of the whitebox approach.
- Requires no assumptions on the system.
- Flexibility of the blackbox approach.
- Requires less training data than the blackbox approach.

Open System Hamiltonian

$$H = \underbrace{\frac{\Omega}{2}\sigma_z}_{H_0:\text{ intrinsic term}} + \underbrace{F_x(t)\cos(\omega_d t + \phi)\,\sigma_x + F_y(t)\cos(\omega_d t + \phi)\,\sigma_y}_{H_{ctrl}(t):\text{ control term}} + \underbrace{g\beta(t)\sigma_z}_{H_1:\text{ noise term}}$$

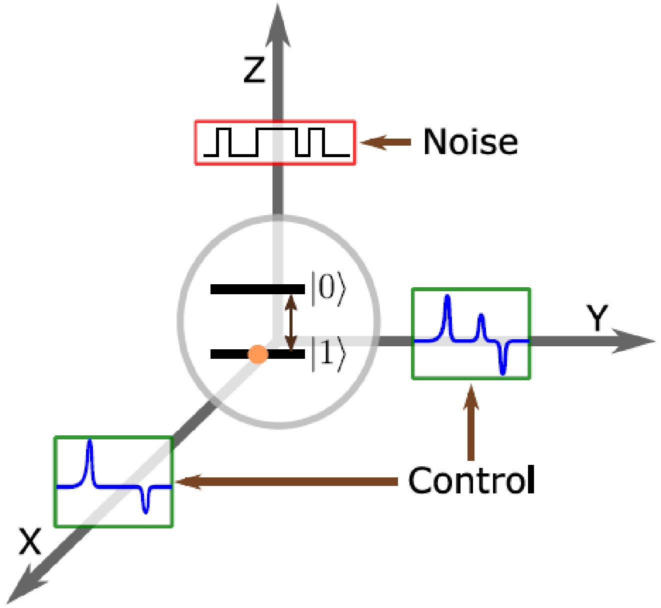
with ω_d drive frequency.

- Noise applied along the z-axis of the Bloch sphere: dephasing noise.
- Control drive applied along the x- and y-axes.

By moving to a suitable rotating frame and applying the rotating wave approximation, with a resonant drive, we get:

$$\tilde{H} = f_x(t)\sigma_x + f_y(t)\sigma_y + g\beta(t)\sigma_z$$

with $f_x(t) = \frac{F_x(t)}{2}, f_y(t) = \frac{F_y(t)}{2}$.



Open System dynamics

The goal

Predict the expectation value $\mathbb{E}[O(T)]_\rho$ of observable O at time T:

$$\mathbb{E}[O(T)]_\rho = tr_S\left[V_O U_{ctrl}(T)\rho(T)U_{ctrl}^\dagger(T)O\right] \quad \text{where} \quad U_{ctrl}(T) = \mathcal{T}_+ exp\left(-i\int_0^T H_{ctrl}(s)ds\right)$$

For classical noise and a traceless observable $V_O = O^{-1}QDQ^\dagger$, where

$$D = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix} \quad , \quad Q = \begin{pmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} e^{i\Delta} & 0 \\ 0 & e^{-i\Delta} \end{pmatrix}$$

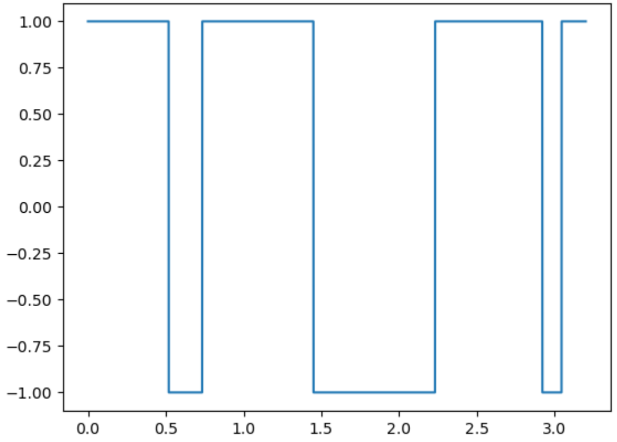
Calculation of V_O is reduced to finding the four parameters $\Delta, \mu, \psi, \theta$.

For $O \in \{\sigma_x, \sigma_y, \sigma_z\}$ and $\rho(0) \in \{|0_x\rangle, |1_x\rangle, |0_y\rangle, |1_y\rangle, |0_z\rangle, |1_z\rangle\}$ calculation of $\mathbb{E}[O(T)]_\rho$ yields tomographically complete information on $\rho(T)$.

Noise models

Random Telegraph Noise (RTN)

- Stochastic, binary, discrete-state process.
- Non-Gaussian, Markovian.

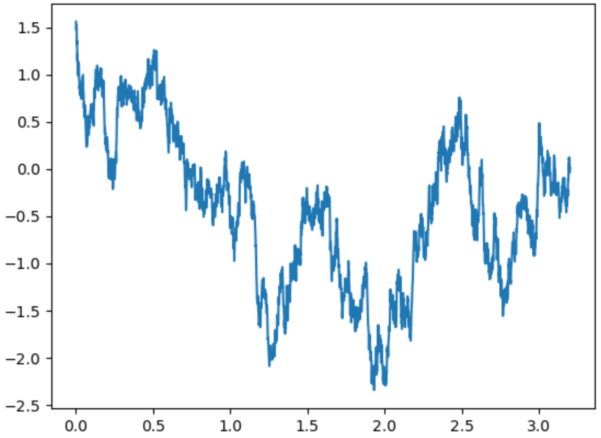


$$\xi(t) = \xi(0)(-1)^{n(0,t)} \qquad \mathbb{P}[n(0,t) = k] = \frac{(\gamma t)^k}{k!} e^{-\gamma t}$$

$$\xi(0) = \pm 1 \qquad \langle \xi(t_1)\xi(t_2) \rangle = e^{-2\gamma |t_1 - t_2|}$$

Ornstein-Uhlenbeck Noise

- Stochastic, continuous, Markovian process.
- Gaussian.
- Mean-reverting.



$$X(t) = X(0)e^{-kt} + \mu\left(1 - e^{-kt}\right) + \sqrt{D}\int_0^t e^{-k(t-t')} dW(t')$$

$$\langle X(t_1)X(t_2) \rangle \to \frac{D}{2k} e^{-k|t_1 - t_2|}$$

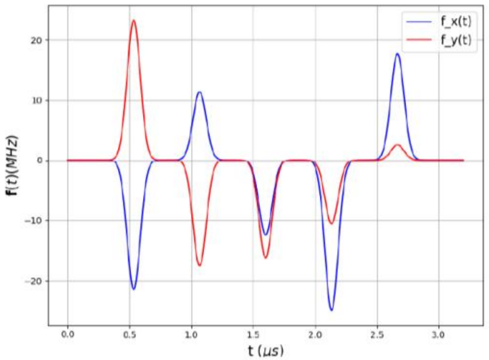
Model training and Optimal Control

Training

- Data produced through Monte Carlo simulations.
- Input: random set of control pulse amplitudes.
- Output: fidelities for a universal set of 1-qubit gates.
- Two-axis control.
- Control achieved through Gaussian pulses:

$$f_\alpha(t) = \sum_{k=1}^{N} A_{k,\alpha}\, exp\left(-\frac{(t-\tau_k)^2}{\sigma^2}\right)$$

- Control parameters: $N = 5$, $\tau_k = \frac{k}{N+1}T$, $\sigma = \frac{T}{12N}$.
- Dataset generated for various values of the coupling strength g.



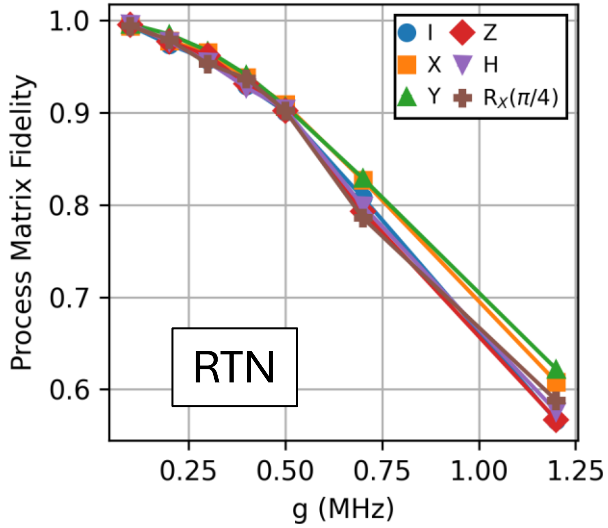
Optimal Control

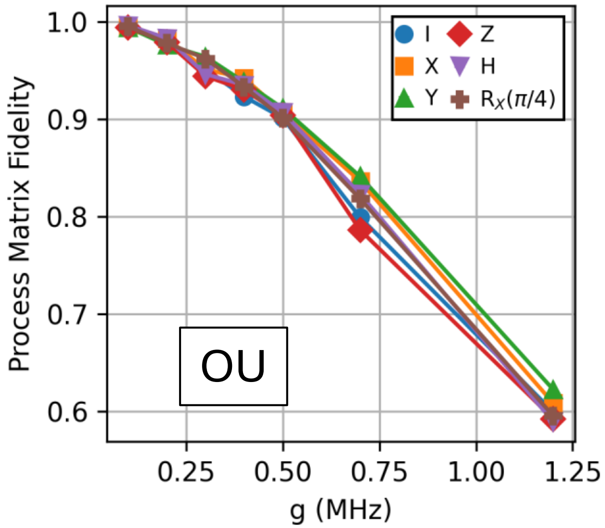
Approach:

- After training, the model is used as a fast dynamics emulator.
- Optimization of pulse parameters $u(t)$ to implement a universal set of single qubit gates.

Optimization criterion:

- Cost functional: $J(u,\Theta;G) = 1 - F(u,\Theta;G)$, where $F = tr(\chi_{actual}^{\dagger}, \chi_{target})$.
- Optimization done using L-BFGS-B method.





Conclusions & Outlook

Summary

- Graybox model accurately emulates qubit dynamics across a wide range of coupling strengths ($g = 0.1 - 1.2$ MHz).
- Gate fidelities above 99% at the lowest considered values of g. Remaining above 90% up to $g = 0.5$ MHz.
- Similarity in results between RTN and OU shows robustness of the model.

Future developments

- Adaptation to real-device constraints.
- Extension to multi-qubit and multi-level systems.
- Application to dynamical decoupling protocols.

Thank you for your attention.

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